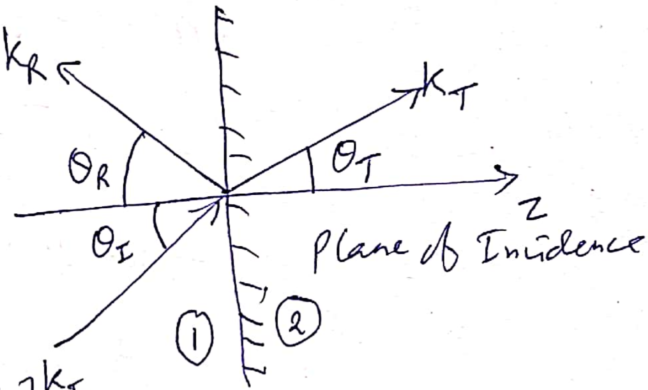
Reflection and Transmission at Oblique Incidence

Normal incidence $\longrightarrow$ Incoming wave hits the interface head-on.

More general case $\longrightarrow$ Oblique incidence
$\hookrightarrow$ Incoming wave meets the boundary at an arbitrary angle θ_I.

Normal incidence $\rightarrow$ Special case of oblique incidence with $\theta_I = 0$



Monochromatic plane wave

$$\left.\begin{aligned} \tilde{E}_I(r,t) &= \tilde{E}_{0_I} e^{i(k_I \cdot r - \omega t)} \\ \tilde{B}_I(r,t) &= \frac{1}{v_I}(\tilde{k}_I \times \tilde{E}_I) \end{aligned}\right\} \quad \text{--(25)}$$

approaches from the left, giving rise to a reflected wave

$$\tilde{E}_R(r,t) = \tilde{E}_{0R} e^{i(k_R \cdot r - \omega t)}, \quad \tilde{B}_R(r,t) = \frac{1}{v_1}(\tilde{k}_R \times \tilde{E}_R) \quad \text{--(26)}$$

and a transmitted wave

$$\tilde{E}_T(r,t) = \tilde{E}_{0T} e^{i(k_T \cdot r - \omega t)}, \quad \tilde{B}_T(r,t) = \frac{1}{v_2}(\tilde{k}_T \times \tilde{E}_T) \quad \text{--(27)}$$

All three waves have the same frequency $\underline{\omega}$

The wave numbers are related by

$$k_I v_1 = k_R v_1 = k_T v_2 = \omega \quad \text{or} \quad k_I = k_R = \frac{v_2}{v_1} k_T = \frac{n_1}{n_2} k_T \quad \text{--(28)}$$

The combined fields in medium (1), $\tilde{E}_I + \tilde{E}_R$ and $\tilde{B}_I + \tilde{B}_R$ must be joined to the fields $\tilde{E}_T$ and $\tilde{B}_T$ in medium (2), using boundary conditions.

They share the generic structure

$$(\ )e^{i(k_I \cdot r - \omega t)} + (\ )e^{i(k_R \cdot r - \omega t)} = (\ )e^{i(k_T \cdot r - \omega t)}, \quad \text{at } z=0 \qquad \text{--- (29)}$$

x, y, z and t dependence $\longrightarrow$ confined to the exponents.

Boundary conditions must hold at all points on the plane $\longrightarrow$ and for all time, these exponential factors must be equal (at z = 0).

Time factors are already equal.

$\hookrightarrow$ transmitted and reflected frequencies must match the incident one.

For the spatial terms, evidently

$$k_I \cdot r = k_R \cdot r = k_T \cdot r, \quad \text{when } z=0 \qquad \text{--- (30)}$$

or more explicitly

$$x(k_I)_x + y(k_I)_y = x(k_R)_x + y(k_R)_y = x(k_T)_x + y(k_T)_y, \qquad \text{--- (31)}$$

for all x and y

Eq^n(31) can hold only if the components are separately eq if $x=0$, we get

$$(k_I)_y = (k_R)_y = (k_T)_y \quad \text{--- (32)}$$

while $y=0$,

$$(k_I)_x = (k_R)_x = (k_T)_x \quad \text{--- (33)}$$

We may as well orient axes so that k_I lies in the xz plane (i.e. $(k_I)_y = 0$); accordingly from eq^n (32) k_R and k_T also.

First Law: The incident, reflected and transmitted wave vectors from a plane → plane of incidence

also includes the normal to the surface (z-axis)

Eq^n (33) implies that

$$k_I \sin\theta_I = k_R \sin\theta_R = k_T \sin\theta_T \quad \text{--- (34)}$$

angle of incidence — angle of reflection — angle of transmission (Refraction)

in view of eq^n (28)

Second Law: The angle of incidence is equal to the angle of reflection

$$\theta_I = \theta_R \quad \text{--- (35)}$$

Law of reflection

Third Law

$$\boxed{\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}} \quad \text{--- 36}$$

Law of refraction → Snell's Law